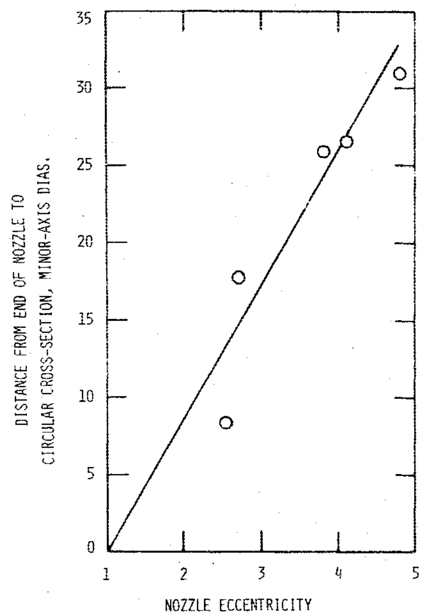

Fig. 5 Distance from nozzle exit (in minor axis diameters) to initial circular cross section of jet, as a function of nozzle eccentricity *e*.

Now we examine the claim, mentioned by Crighton,[3] that the "flat" part of an elliptical aircraft nozzle jet radiates less noise than a similar circular jet. We can see no instability patterns, or lack of them, on the surfaces of the jet to explain a lower radiation from the wide part of the jet, and indeed Maestrello and McDaid[8] found very little difference in near-field noise emission from various sides of a rectangular jet. What could be happening, however, is the 90-deg shift in major axis shortly after leaving the aircraft, which then presents a smaller width radiating source to observers on the ground than would a circular jet of the same power. This then could explain the possible noise benefits of elliptical nozzles, although more elaborate theories have been proposed.[9]

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Pseudo-transonic Equation with a Diffusion Term

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Introduction

THE purpose of this Note is to show how Hayes'[1] pseudo-transonic equation for supersonic flow, when modified by a streamwise diffusion term, can be transformed into Burgers' equation. The solution of this equation reduces to that of Whitham[2] in the limit of vanishing viscosity. The role of diffusion near shocks is well known, and in this Note certain simplifying features introduced by viscosity are discussed.

Analysis

Ackeret's classical solution is based on an approximate linearized equation that is a proper first approximation near the surface, but which breaks down at large distances from the airfoil. For example, classical theory fails to predict the bending of Mach lines that occurs in reality, and the subsequent shock formation and decay. This failure arises from the neglected cumulative effects of locally small disturbances that grow to first order over large distances. It is now well known that the nonuniformity of linear theory arises only from the neglect of the "pseudo-transonic" term $(\gamma+1)M_\infty^4 \varphi_x \varphi_{xx}$ in the equation for the disturbance velocity potential φ, where γ is the ratio of specific heats, M_∞ is the freestream Mach number, x is the streamwise coordinate and y is to be the stream-normal coordinate; the contribution of all other nonlinear terms is uniformly of second order. This pseudo-transonic term has a first-order cumulative effect and must be retained in addition to the usual linear ones in seeking a uniformly valid first approximation. To the same order of accuracy, the tangency condition can be imposed on the axis, and this completes the usual nonlinear, supersonic, *inviscid* formulation.[1,2]

In this Note, the physical problem is considered as a limit of the diffusive system

$$\delta\varphi_{xxx} + (1-M_\infty^2 - (\gamma+1)M_\infty^4\varphi_x)\varphi_{xx} + \varphi_{yy}(x,y) = 0 \quad (1a)$$

$$\varphi_y(x,0) = \epsilon T'(x) \quad (1b)$$

$$\varphi = 0 \text{ upstream} \quad (1c)$$

where $\delta > 0$ is a small diffusion coefficient, ϵ is the thickness ratio, and T' is a normalized slope. It is interesting that Eq. (1) is just the "viscous transonic equation" as derived by Cole,[3] although in the present application we are dealing with purely supersonic applications. In the transonic case, Eq. (1) is obtained through a special limiting process taking into account the effect of compressive viscosity at sonic lines and shocks, where the coefficient of φ_{xx} normally would vanish in low-order theory; to the order considered, the effects of rotationality introduced by viscosity and shock curvature can be ignored. We can expect solutions of Eq. (1) to reduce in the limit of $\delta \to 0$ to solutions of the inviscid formulation; that they actually do in the transonic case was demonstrated by

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Sichel[4] for planar and axisymmetric nozzles. In both cases the solution provided a viscous, shock-like transition from an inviscid, supersonic, accelerating flow to an inviscid, subsonic, decelerating flow. The addition of viscosity here enables us to treat the shock structure as well, but the structure obtained below is only a rough model of the real flow.

For simplicity, consider a symmetric airfoil and introduce the following transformation of coordinates. Two directions appear in the problem, that of the freestream and that of the outgoing freestream Mach lines. The airfoil lies nearly along the former and the wave pattern nearly along the latter. Thus the solution can be expressed more naturally in terms of the oblique coordinates $\xi = x - (M_\infty^2 - 1)^{1/2} y$ and $\eta = (M_\infty^2 - 1)^{1/2} y$ as first introduced by Hayes.[1] Coordinate transformations for $\varphi(\xi,\eta)$ then produce terms like φ_η and $\varphi_{\eta\eta}$ which represent truly second-order effects which can be dropped.[1] When these terms are discarded in the equation and in the surface boundary conditions, and new barred variables are defined by $\eta = \bar{\eta}$, $\varphi_x = \varphi_\xi = u = 2^{1/2}\ (M_\infty^2 - 1)^{1/2}\ \bar{u}/[(\gamma+1)M_\infty^4]$ and $\xi = \bar{\xi}/(2(M_\infty^2 - 1))^{1/2}$, the formulation as given in Eq. (1) becomes

$$\bar{u}_{\bar{\eta}} + \bar{u}\bar{u}_{\bar{\xi}} = \delta \bar{u}_{\bar{\xi}\bar{\xi}} \tag{2a}$$

$$\bar{u} = -\epsilon \frac{\gamma+1}{\sqrt{2}} \frac{M_\infty^4}{M_\infty^2 - 1} T' \left\{ \frac{\bar{\xi}}{\sqrt{2(M_\infty^2 - 1)}} \right\} \equiv F(\bar{\xi}) \text{ at } \bar{\eta} = 0 \tag{2b}$$

$$\bar{u} = 0 \text{ upstream} \tag{2c}$$

This is just the initial value problem for Burger's equation, for which the closed-form solution is known.[5] The solution is simply

$$\bar{u}(\bar{\xi},\bar{\eta};\delta) = \frac{\int_{-\infty}^{\infty} \frac{\bar{\xi}-\lambda}{\bar{\eta}} e^{-G/2\delta} \mathrm{d}\lambda}{\int_{-\infty}^{\infty} e^{-G/2\delta} \mathrm{d}\lambda} \tag{3}$$

where

$$G(\lambda;\bar{\xi},\bar{\eta}) = \int_0^\lambda F(\lambda')\mathrm{d}\lambda' + \frac{(\bar{\xi}-\lambda)^2}{2\bar{\eta}} \tag{4}$$

and can be readily re-expressed in physical coordinates.

Summary

It is clear that solutions of Eq. (2) tend to those of inviscid theory as $\delta \to 0$ by virtue of the known properties of Burger's equation. The addition of viscosity here, in fact, actually makes the mathematical problem simpler. In the usual inviscid formulation the solution for u is implicit, whereas that given above is explicit (this representation is useful in many applications). Furthermore, shocks must be constructed in the far field to prevent multi-valued solutions; viscous diffusion in the physical problem here automatically gives a single-value solution. In addition, the present approach furnishes a model for the shock structure and decay that, for supersonic flows, is correct in the limit $M_\infty \to 1$. As in the inviscid theory, simple surface-slope discontinuities are permissible, as are slightly rounded leading edges, provided that the solution is used in the distant far field. The present results are exactly the same as those in Whitham's theory, and the approximate structure of the shock region is that determined on the basis of Burger's equation.

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Large-Amplitude Free Vibration of Tapered Hinged Beams

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Nomenclature

$A(\xi), A_0$ = area at any station and reference station, respectively
$a(\xi)$ = nondimensional area variation
$I(\xi), I_0$ = moment of inertia at any station and reference station, respectively
$i(\xi)$ = nondimensional variation of inertia
E = modulus of elasticity
$F(t)$ = time function
$F_q(\xi)$ = summed up inertial forces at station
L = length of one-half of beam
m_0 = mass per unit length of beam at reference station
N, N_m = axial force at any station, at $\xi = 1$
Q, Q_m = shear force at any station, at $\xi = 1$
$q(\xi)$ = transverse inertial force per unit length
U, V = displacements of a point on neutral axis in x, y directions
u, v = U/L, V/L
x, y = coordinate system
α = taper parameter
$\epsilon, \epsilon_m, \bar{\epsilon}$ = axial strains
ξ = x/L, nondimensional coordinate
θ, θ_0 = slope at any station, at reference station
λ = nondimensional eigenvalue-like quantity
ρ = radius of gyration at reference station
ω = quantity characterizing vibration

I. Introduction

THE large-amplitude free flexural vibrations of uniform beams have been studied.[1-4] Raju et al.[5] solved the corresponding problem for tapered beams using a one-term trigonometric or polynomial mode shape and a Galerkin technique. However, the use of one mode shape for all tapers and amplitudes of vibration will be quite inaccurate. In this Note, we examine this problem using a simple, numerically exact, successive integration and iterative technique, together with an eigenvalue quantity arising from certain definitions of the time function. The actual nonlinear equilibrium equations (e.g., specification of loads in terms of the deformed configuration) and the exact nonlinear expressions of curvature are used. No assumptions are made as to the constancy of axial force. The hardening effect of nonlinearity is interpreted

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